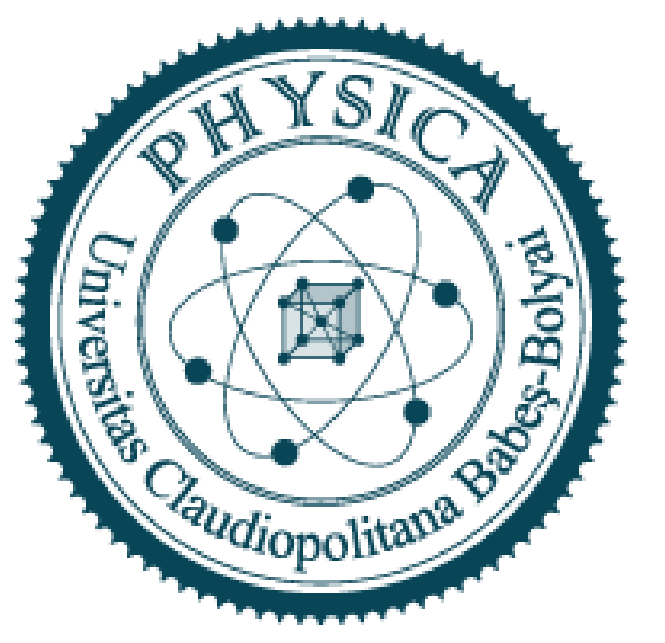




Wealth inequalities in different socio-economical situations. Exhaustive data and a general modelling framework.



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Abstract

Quantifying socio-economic inequalities through wealth or income is a popular topic both in economics and econophysics. These studies were catalysed in the early IX. century by the famous work of Pareto, who recognised the highly unusual heavy-tail shape of the wealth and income distribution statistics. Since then many experimental and theoretical investigations were done to unveil interesting universalities. Modelling needs good quality and reliable experimental data, however wealth is not easy to measure directly and momentarily we lack exhaustive data for well delimited communities. Our study intends to fill this niche, by offering precious wealth data for all households in a commune from Transylvania (Romania) for three very different economic situations: (1) socialism before the collectivization of lands -1961-, (2) the last year of the communist regime -1989- and (3) the present situation, 2021, 32 years of free market economy after the fall of communism. Wealth is estimated using agricultural records and taxation data. For modelling we consider the stationary limit of an analytically solvable evolution equation with local growth and reset processes. Such an approach was used recently in describing in a unified manner wealth distribution in modern societies [1]. With realistically chosen growth and reset rates the model successfully describes the experimentally observed distributions and inequality measures for all three different economic periods [2].

Background and Motivation

- The interest of physicists toward socio-economic inequalities was detonated by the discovery of the fat-tailed nature of wealth and income distributions by Vilfredo Pareto [3, 4, 5].
- Inequalities in the society can be quantified by measuring income and wealth.
- Wealth is difficult to measure because it is composed of different types of valuables.
- The wealth estimation methods existing in the literature might be biased. – we propose a more robust wealth estimation method based on exhaustive wealth data
- The influence of the country's economic orientations on the wealth distribution in a small community is also a fascinating question.
- It is also fascinating to see, how the country's economic orientation reflects on the distribution of wealth in a small community.

Processed data and methodology of wealth estimation

- exhaustive wealth datasets from a commune (Sâncraiu, 46°49'47"N 22°59'18"E) from Romania
- agricultural records and taxation data are processed for three years, that are radically different from economic point of view
- 1961 - communist regime, collectivization of the agriculture destined land
- 1989 - last year of the communist regime, most of the private property is abolished
- 2021 - after 31 years of free market economy

One page of the agricultural register from 1989.

- the wealth of each household was calculated considering 10 different sets of category weights (P_i), which guarantees the unbiased nature of the estimation

$$W_{\text{household}} = \frac{\sum_{i=1}^{N_C} C_i \cdot P_i}{\sum_{i=1}^{N_C} P_i}$$

weight of i^{th} cat.

Local Growth and Global Reset (LGGR) process

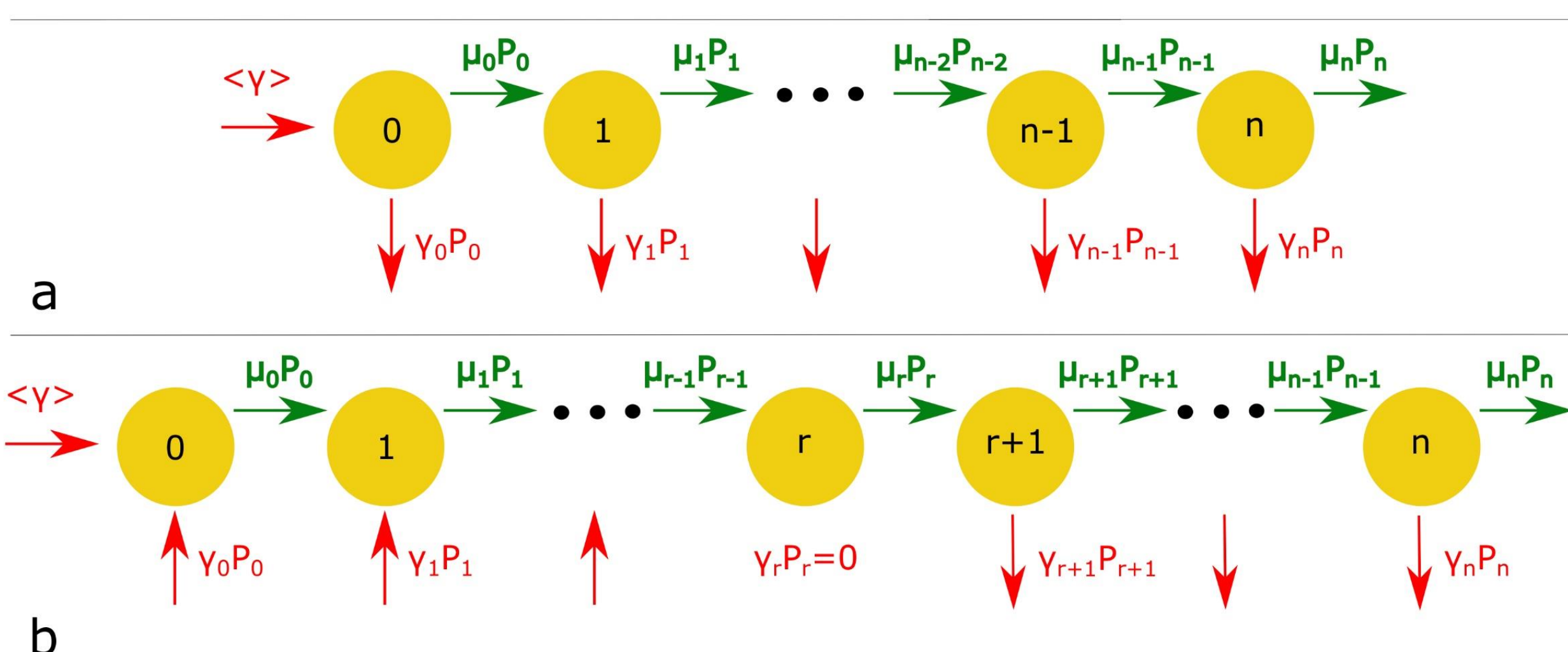
- a master equation based framework
- appropriate to demonstrate the underlying dynamics of wealth and income distributions [1, 2, 6, 7]

Discrete case: $\frac{dP_n(t)}{dt} = \mu_{n-1}P_{n-1}(t) - \mu_n P_n(t) - \gamma_n P_n(t) + \delta_{n,0} \langle \gamma \rangle (t).$

Continuous case:

$$\frac{\partial \rho(x, t)}{\partial t} = -\frac{\partial}{\partial x} [\mu(x) \rho(x, t)] - \gamma(x) \rho(x, t) + \langle \gamma(x) \rangle (t) \delta(x)$$

stationary solution $\rightarrow \rho_s(x) = \frac{\mu(0) \rho_s(0)}{\mu(x)} e^{-\int_0^x \frac{\gamma(u)}{\mu(u)} du}$



Representation of the growth and reset process: (a) the process with a positive reset rate, (b) the considered reset rate can be both positive and negative.

Acknowledgement

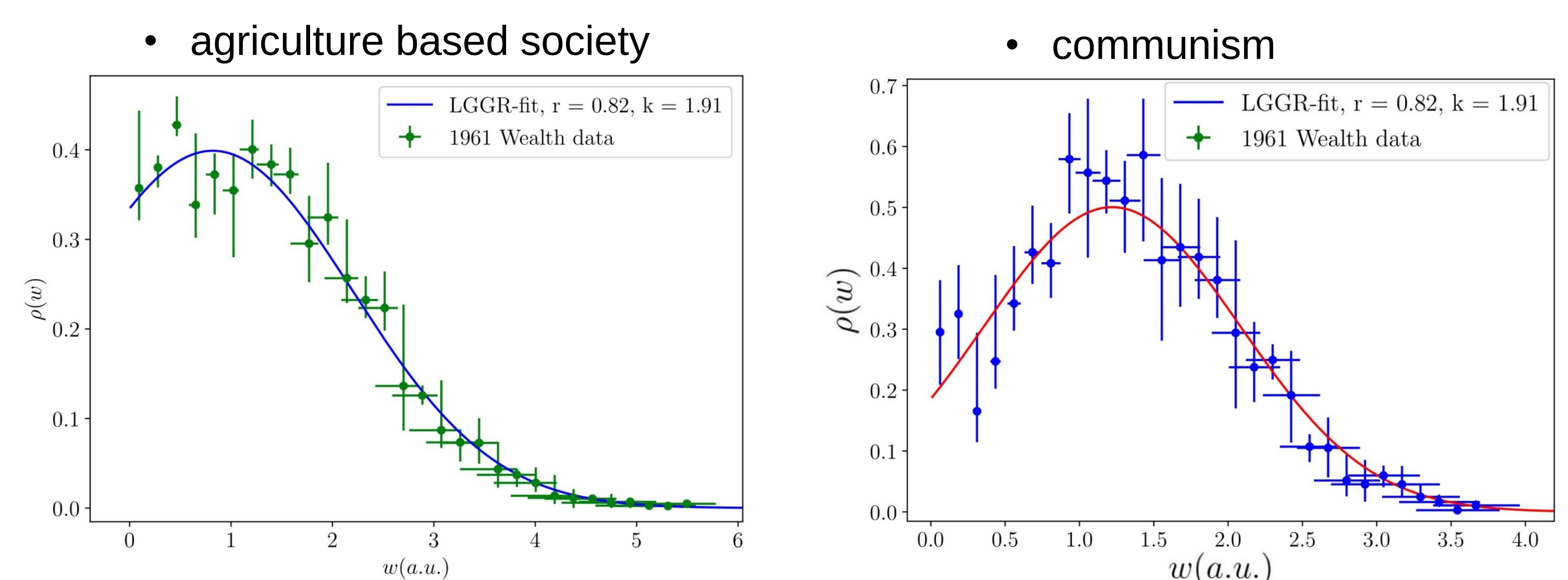
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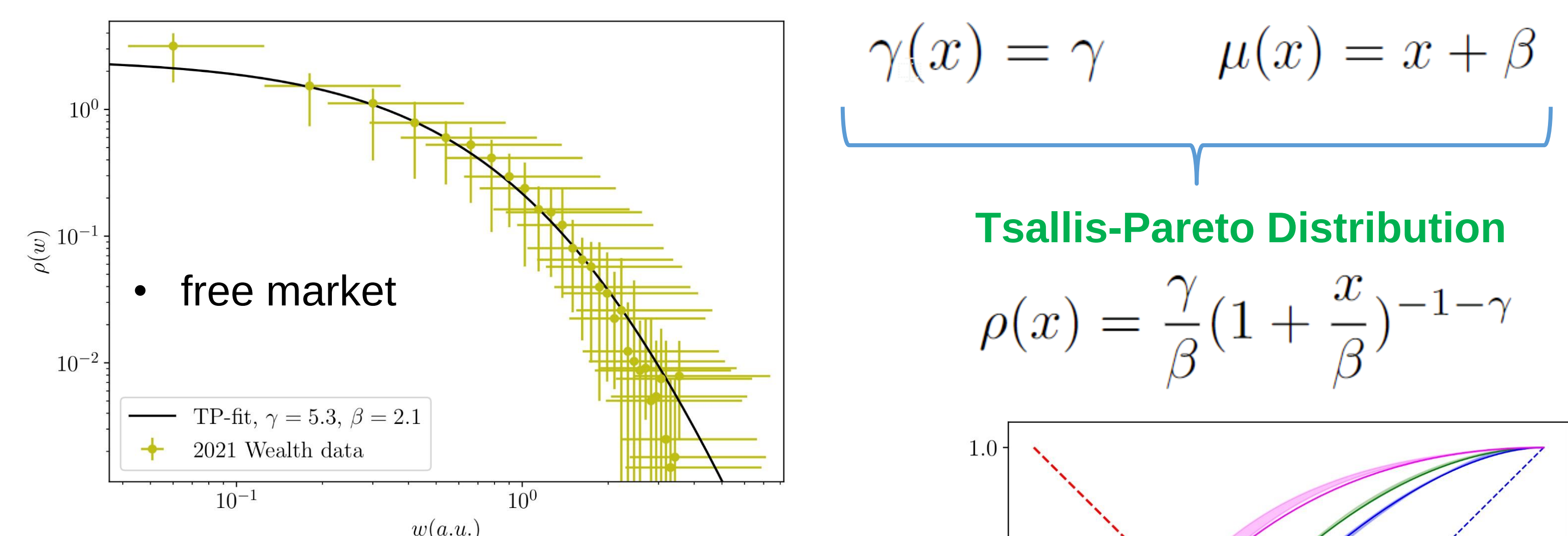
Wealth distributions

$$\mu(x) = k \quad \gamma(x) = x - r \quad \rho(x) = \sqrt{\frac{2}{k\pi}} e^{-\frac{x(x-2r)}{2k}} \left[\text{erf}\left(\frac{r}{\sqrt{2k}}\right) + 1 \right]^{-1}$$

Normal Distribution



Wealth distributions for agriculture based society (1961) (left) and communist society (1989) (right). The error bars indicate the accuracy of the wealth estimation method.



Wealth distribution in case of the free market economy (2021).

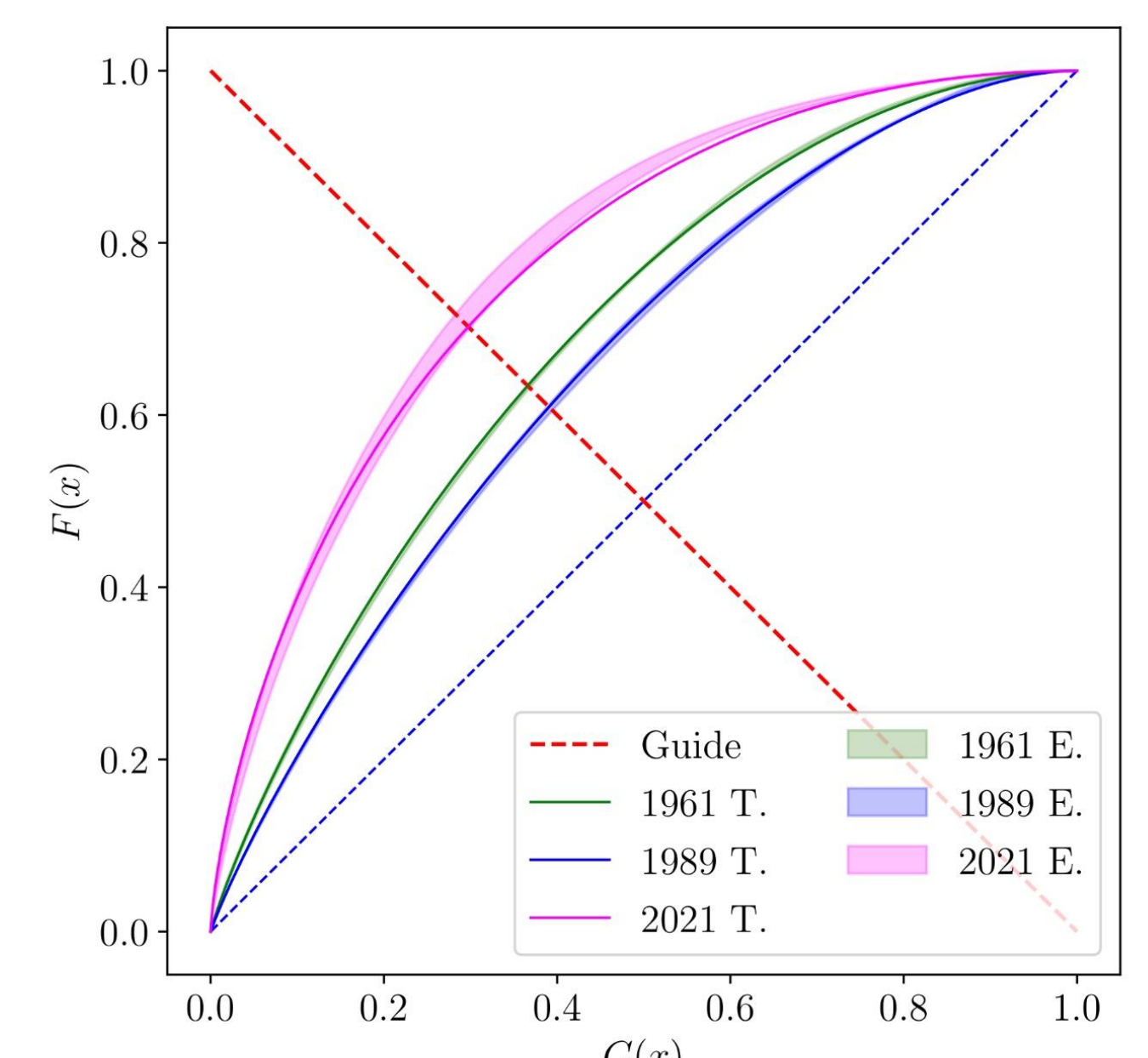
Calculated inequality indexes

- the **Gini coefficient**, the **Lorenz curve** and the **Pareto point** were compared

$$G = \frac{1}{2n^2 \langle x \rangle} \sum_i^n \sum_j^n |x_i - x_j|$$

$$C(x) = \int_x^\infty \rho(w) dw$$

$$F(x) = \frac{1}{\langle x \rangle} \int_x^\infty w \rho(w) dw$$



The Lorenz curves and Pareto points (P) calculated for the three studied years. The shaded region indicates the accuracy of the wealth estimation method.

Year	G		P	
	Exp.	Theo.	Exp.	Theo.
1961	[0.377;0.379]	0.378	[0.366;0.368]	0.367
1989	[0.304;0.315]	0.312	[0.390;0.395]	0.391
2021	[0.543;0.579]	0.552	[0.282;0.299]	0.298

Inequality measures for the studied years: the Gini (G) and Pareto-point (P).

Conclusions

- starting from exhaustive information about the owned valuables of households we proposed an unbiased method for wealth estimation
- using the LGGR framework we successfully described the wealth distributions in three different economic situations - the used rates reflect the economic policies of the studied years
- we demonstrated the effect of the economic orientation of the country on the distribution of wealth – these results are in agreement with the economic ideology

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